Level 1: The Number Partner
http://nzmaths.co.nz/resource/number-partner?parent_node=

The Number Partner

Purpose:

This unit uses one of the digital learning objects, the number partner, to support students as they investigate possible pairings for numbers from 10 to 30. It is suitable for students working at stage 3 - 4 of the number framework. It includes problems and questions that can be used by the teacher when working with a group of students on the learning object, and ideas for independent student work.

Achievement Objectives:

Achievement Objective: Use a range of counting, grouping, and equal-sharing strategies with whole numbers and fractions.
Achievement Objective: Know groupings with five, within ten, and with ten.

Specific Learning Outcomes:

identify number pairs that sum to numbers from 10 to 30

use number pairs to solve addition and subtraction problems

Description of mathematics:

The strategy section of the New Zealand Number Framework consists of a sequence of global stages that students use to solve mental number problems. On this framework students working at different strategy stages use characteristic ways to solve problems. This unit of work and the associated learning object are useful for students in transition between stages 3 and 4 of the Number Framework, moving from Counting from One by Imaging to Advanced Counting. This transition involves students moving from counting all the objects when joining 2 sets, to starting the count from the highest number and then "counting on" to find the total number. For example, when adding 5 and 3 students working at stage 3 will count all eight objects starting from 1, while students at stage 4 start counting at 5 and add the extra three, (5...6,7,8) to get the answer. The Number Framework also includes a knowledge component which details the knowledge students will need to develop in order to progress through the strategy stages of the framework. This unit can be used to develop students' knowledge in the areas of grouping and basic facts; in particular knowledge of the pattern of teen numbers, groupings within 20 and doubles with a sum greater than 10 can be built up.

The learning object, the number partner, can be accessed from the link below, using your school's username and password:

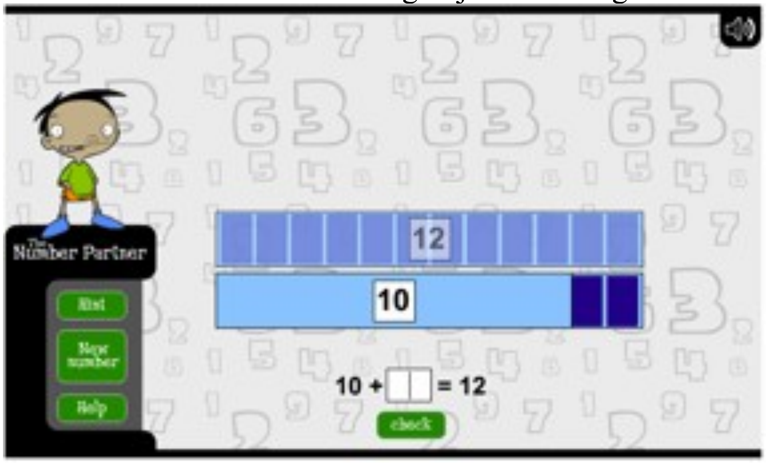
- The number partner

Click for more information about the learning objects and who can register to use.

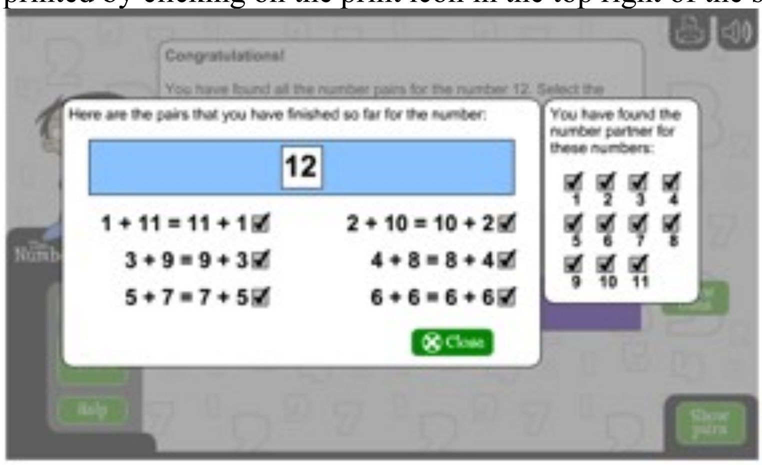
Activity:

1. Show students the learning object and introduce them to Alex, explain that he is going to help them break numbers into pairs.

2. Enter a number into the learning object. 12 is a good number to start with.



3. Show the students that the top bar is the number to be broken up into pairs and confirm that it is 12 by counting the bars that represent ones.
4. Explain to the students that part of the number is hidden under the light blue bar and that the number on the bar shows how many are hidden. Confirm by counting along the top bar that it is 10 that are hidden in the first example.
5. Ask the students "Ten and how many more are twelve?" Count the ones to confirm the answer and enter this into the box. Check this using the green button.
6. Complete another number pair for 12 in the same way. Questions that can be used include: *9 and how many more are 12? What is the number partner that will join with 9 to make 12?*
7. Click the "next" button to find further number pairs for 12. Show the students how the slider works, including the button that can show and hide the ones.
8. Ask the students to choose a number to work with and enter this into the learning object. Have the students identify the number partner for this example, initially with the ones hidden, then check this answer by showing the ones and counting them to confirm the result.
9. Continue for several more number partners of 12 then show the students the "show pairs" button which keeps a track of the pairs that have been found.
10. Continue to find all the number pairs for 12. The number pairs found can be printed by clicking on the print icon in the top right of the screen.



Explain to the students that they are going to use the learning object to help them work out how many jumps Freddy the frog will have to make to his friend's house.

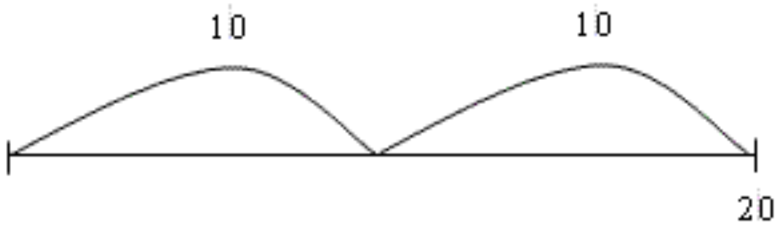
1. First select the distance between Freddy's house and his friend's house in metres and enter this as a new number to be broken into pairs. 15 would be a good example to use.
2. Explain that Freddy is going cover the 15 metres to his friend's house in 2 jumps. Ask the students how many metres he will cover in his first jump and enter this into the number partner.
3. Use the number partner to help work out how many metres he will have to cover in his second jump to arrive exactly at his friend's house.
4. Use the number partner to work out the other combinations of jumps Freddy could do to get to his friends house 15 metres away, using 2 jumps each time.
5. As students work encourage them to count on from the highest number when working out the second number in a pair.
   We know he's already jumped 11 metres, let's start counting from there. What's the next number after 11? (This is most easily asked when the ones in the bottom bar are hidden.) *We know 11 and 4 are number partners to make 15. Let's check that 11 and 4 are 15: 11...12, 13, 14, 15.* (Most easily asked when the ones are shown in the bottom bar).
6. Develop the idea that the order of numbers does not matter in addition. For example, adding 11 and 4 is the same as adding 4 and 11. Clarify that counting on from the largest number is the most efficient strategy.
   If we are adding 11 and 4 what is the best number to start counting from? Why?
7. Repeat the problem using different distances between Freddy's house and his friend's place.

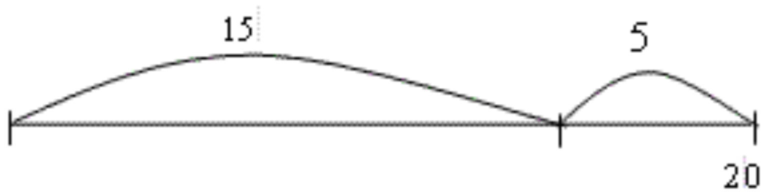
Students working independently with the learning object

Use one of the contexts below to set problems for the students to solve independently, either on their own or in pairs, using the learning objects.

- Savings, for example:
  You are saving for a skateboard which costs $20. If you have saved $8 so far, how much more do you need to save before you have enough for the skateboard? What if have $13 saved, how much more do you need?
- Traveling, for example:
  If you are going to visit your Nana who lives 26 km away and you have traveled 7 km already, how much further do you need to travel? What if you have already traveled 13 km?
- Monkeys in trees, for example:
  If there are 14 birds playing in two trees and 8 of the birds are in one tree, how many are in the other tree? What if there are 10 birds in the first tree?

Once students have solved the problem ask them to draw a diagram to show how they used the learning object in their solution. Using a number line in their representation would be useful but encourage students to use a variety of methods to record their thinking. For example:





How did you use the number partner?
What numbers did you use?
What numbers did you need to count to work out the answer?
How could you show what you did in a diagram?

When all students have described their solutions in a diagram, reassemble as a group and have students describe their solutions to each other using their diagrams.

Students working independently without the learning object

Independent activities that develop the same concepts as the learning object include:

- Students play a game of "How Many are Hiding?" with a partner. To play this game one student assembles a collection of objects, and hides some of them under a container or a piece of card. They tell their partner how many there are in total and their partner then works out how many are hiding. For example, "I have 26 buttons, and there are 12 in this pile. How many are hiding?"
- Use multilink cubes to make up "packets of lollies" using 2 colours. "If there are 12 lollies in a packet and there are 8 red ones, how many green ones are there? What if there are 6 red ones?" Record all the possible combinations of the two colours of lollies.

Home Link:

Dear Family,

This week at school we have been thinking about number pairs. Part of what we have been doing is playing a game called "How Many are Hiding?" To play this game one person counts out a collection of objects, and hides some of them under a container or a piece of card. They then ask the other person to work out how many are hiding. For example, "I have 26 buttons, and there are 12 in this pile. How many are hiding?" It would be great if you could play this game together at home. Thanks

In this unit students explore different ways to communicate and explain adding numbers within ten. The representations included are number lines, set diagrams, animal strips and tens frames.

Level 2: Smart Doubling
http://www.nzmaths.co.nz/resource/smart-doubling?parent_node=

Smart Doubling

Purpose:

In this unit students are encouraged to abandon counting methods for adding and subtracting in favour of more complex but more efficient mental strategies using known doubles facts to derive the answers to addition problems.

Achievement Objectives:

Achievement Objective: Use simple additive strategies with whole numbers and fractions.
Achievement Objective: Know the basic addition and subtraction facts.

Specific Learning Outcomes:

be able to recall instantly doubles from 1 + 1 to 9 + 9

use known doubles facts to work out addition problems mentally using part/whole reasoning

Description of mathematics:

In this unit students who currently use "counting on" or "counting down" methods to solve addition and subtraction problems are encouraged to use doubles and "part/whole" mental methods to speed up mental computation.

Examples of "counting on" or "counting down" methods are

- The students work out 16 – 13 by starting at 13 and count 14, 15, 16; they know the answer is 3.
- The students work out 29 + 4 by counting 30, 31, 32, 33; they know the answer is 33.

Examples of "part/whole" methods for doubles are:

- The students work out 8 + 6 by removing 2 from the 8 to leave 6, then 6 + 6 =12 from a known doubles fact, and then add the removed 2 to 12 to give 14.
- The students work out 26 + 25 by adding 25 + 25 to give 50 and adding 1 to give 51.

It is desirable for students to move from counting methods to part/whole methods as counting methods are too slow for larger numbers. For example, students who attempt to work out 35 + 36 by "counting on" will soon lose their way.

The transformation from a counting method to the more abstract part/whole methods may prove difficult for students. The idea of breaking numbers into suitable parts to help solve the problem is not initially obvious and may take considerable effort for the students to understand the principle. For example students might model this problem on counters: Jean has 7 red sweets and buys 8 green sweets at the shop. How many has she got altogether?

A green sweet is removed to leave 7 in one pile and 7 in the other. Students, because they know their doubles, now say 7 + 7 = 14. Critically they must now add the removed sweet to 14 to give 15.

While this process often makes sense for students when materials are present they may experience difficulty when the materials are removed and they are asked to imagine the process. For example, to work out 9 cakes plus 8 cakes the students are asked to "see" without materials, that 9 contains 8 and 1, then to understand they use the known double 8 + 8 = 16, then they add the 1 to give 17.

The step to being able to ignore materials and working out addition problems by imaging is an important step along the way to being able to quickly and reliably use a "near doubles " strategy. Persistence on the part of the teacher is needed to help students make this transformation from needing materials to be able to image the answer.

Required Resource Materials:
Sets of counters in two different colours
Activity:

Getting Started

1. Check that the students know their doubles facts before starting. If the students do not know all the facts identify which ones they do know work from these ones.
2. Pose addition problems that are near doubles, that is to say the two numbers to be added are almost equal. Using counters encourage the students to solve these problems by removing or adding counters to make the piles equal, then adding or removing to finish the problem.

   Problem

   Sarah has 6 sweets and Peter has 8 sweets. (Students model this with counters.)
   Sarah complains it is not fair because Peter has got more sweets.
   Peter gives a third student some of his sweets so Sarah and Peter are now equal (Students act this out with counters).
   How many sweets does Peter give Roger? (2)
   How many sweets do Sarah and Peter have now? (6+6=12)
   How many sweets are there altogether (12+2=14)

   Another strategy for solving the problem is to make Sarah's and Peter's piles equal (7+7=14). It usually takes time for the students to understand that Peter needs to give Sarah one sweet. The usual initial response is for the students to say that Peter should give two to Sarah.

3. Pose another doubles problem. This time encourage the students to work out answers without materials by using imaging the counters or other mental processes.

Exploring

Over the next 2-4 days the students work with a partner or in a small teaching group to solve problems involving near doubles. As the students solve the problems they are encouraged to share their strategies with others.

1. Pose number stories for the students to solve using, for example, 6 + 7, 5 + 7, 9 + 8, 8 + 6.
2. Encourage the students to use one or both of the strategies discussed on the previous day:

- Students use materials to solve them using the technique of removing some counters to make the two piles equal, using a known doubles fact then adding back the removed counters.
- Students use materials so that each pile has the same amount.

3. As the students become confident with the solution of the problems using materials encourage them to adjust the numbers mentally.
   June has $18 and Mary has $22. Mary kindly agrees to give Mary some of her money so they will have equal amounts. How much money does Mary give June? How much money do they have altogether?
4. Increase the size of the numbers as the students become confident with making mental adjustments with the small numbers. Pose number stories using numbers such as: 21 + 19, 28 + 32, 49 + 51, 23 + 27, 48 + 22, 79 + 11.
5. Encourage the students to adjust the numbers to make whole numbers of tens. (The numbers have been specially selected so that after adjusting to make the numbers equal the answer can be found by adding the relevant number of tens.)
6. For the "experts" pose number stories using these numbers: 102 + 98, 97 + 303, 298 + 102, 499 + 1001

Reflecting

At the end of each session ask volunteers to explain their working and thinking to the rest of the class.

Home Link:

Dear Family

At school this week we are using our knowledge of doubles to solve number problems. Ask your child to explain their thinking as they solve these problems.

Doubling Along

To work out 9 + 8 Jill says 8 + 8 = 16 and adds 1 to give 17.
Use Jill's method to work out these:

5 + 4 = ☐	6 + 5 = ☐	8 + 7 = ☐
11 + 10 = ☐	11 + 12 = ☐	6 + 7 = ☐

To work out 8 + 12 Briony takes 2 away from the 12 to leave 10 and adds this 2 to the 8 to give 10. So 8 + 12 is the same as 10 + 10 = 20

Use Briony's method to work out these:

4 + 8 = ☐	7 + 9 = ☐	9 + 11 = ☐
7 + 13 = ☐	7 + 5 = ☐	12 + 8 = ☐

To work out 28 + 22 Ian takes 2 away from the 22 to leave 20 and adds this 2 to the 28 to give 30. So 28 + 22 is the same as 30 + 20. So the answer is 50.

Using Ian's method to work out the following sums forces students to use mental strategies for addition and subtraction problems:

26 + 34 = ☐	48 + 52 = ☐	79 + 21 = ☐
27 + 73 = ☐	27 + 33 = ☐	22 + 38 = ☐

Level 2: Make a Ten
http://www.nzmaths.co.nz/resource/make-ten?parent_node=

Make a Ten

Purpose:

This unit follows naturally from the **Smart Doubling [6]** unit.
In this unit students are encouraged to further develop part/whole mental methods by using the strategy of "make a ten".

Achievement Objectives:

Achievement Objective: Use simple additive strategies with whole numbers and fractions. [7]
Achievement Objective: Know the basic addition and subtraction facts. [8]

Specific Learning Outcomes:

demonstrate automatic recall of all pairs of single digit addition facts whose total is 10 or less

use the mental strategy "make a ten" for addition problem

use the most efficient mental strategy for a given problem.

Description of mathematics:

In this unit students are encouraged to add to their use of part/whole with doubles by using "make a ten" methods.

Examples of "part/whole" methods using make a ten:

The students work out 8 + 5 by removing 2 from the 5 to leave 3, add this 2 to the 8 to make 10 then add the 3 to the 10 to give 13.

The students work out 38 + 8 by removing 2 from the 8 to leave 6, add the 2 to the 38 to give 40, then add 6 to the 40 to give 46.

It is desirable for students to move to part/whole methods as counting methods fail for larger numbers. For example, a student who attempts to work out 36 + 46 by counting on will soon lose their way, whereas the part/whole thinker could solve this by adding 30 and 40 to give 70, then add 6 and 6 to get 12 then add 12 to 70 to get 82.

Students who have successfully understood the part/whole methods in the Smart Doubling unit will have little trouble learning the Make a Ten part/whole strategy. Teachers should expect those students who failed to make the part/whole connections with doubles to also struggle with this unit. It may be best not to introduce this unit until the students understand the doubling strategy.

Required Resource Materials:

Sets of counters
Empty tens frames with squares large enough to contain counters
Tens frames (Material Master 4-6)

Activity:

Getting Started

1. Check the students' knowledge of the addition facts that add to ten facts before starting. This unit builds on the students' knowledge of the known facts of ten, i.e., 1 + 9 = 10, 2 + 8 = 10, 3 + 7 = 10, 4 + 6 = 10, 5 + 5 etc.
2. Also check that the students understand the " teen" numbers. For example, show the students a ten and a four on tens frame as shown. They should respond that 10 + 4 is 14 without needing to count-on by ones

3. Pose an addition problem where one of the numbers is just under 10:
   Bill has 9 pink sweets and 6 yellow sweets. How many sweets does he have altogether?
   Ask the students to model this with counters on tens frames.

4. Ask the students to think about ways that they could make this problem quick (or easy) to solve. Encourage the students to think of ways that do not involve counting on by ones.

   For example, Bill moves a yellow sweet to the pink sweets. Now ten and five is shown.

It is important to discuss the fact that the answer to 9 + 6 is the same as the answer to 10 + 5. Without this realisation students will not be able to make progress into mental number processes.

(Note: Although the problem can of course be correctly solved by counting-on the aim of the lesson is to encourage the development of part/whole mental strategies.)

Exploring

Over the next 3-4 days the students are encouraged to use "Make to ten" strategies for solving number story problems.

1. Make up number stories for 9 + 7, 5 + 8, 9 + 8, 7 + 6. Students use counters on empty tens frames and use the technique of filling one of the frames up to show ten counters.
2. Students now use the pre-printed tens frames.
   Moana has 7 oranges and 9 apples. How many pieces of fruit does she have altogether?
   Ask the students to show 7 + 9 with a pre-printed 9 card and a pre-printed 7 card. Students imagine moving 1 out of the 7 to leave 6, and adding 1 to the 9 to create 10. This action of imagining pushes the student towards understanding the movement of numbers around within a problem as a key to mental processing.
3. Only you, as the teacher, have the pre-printed tens frames. Hold the cards so that only you can see the dots and the students have to imagine what you are seeing. Prior to giving the students addition problems ask the students to first imagine single numbers.
   I can see the 8 card. What does it look like?
   Some students will see the 5 + 3, others will "see" the two spaces.
4. Next pose addition problems, which encourage the students to imagine the tens frames that only you can see. For example:
   I can see 8+ 6. How could I move the dots to work that out 8 + 6?
5. The next step in the progression involves **no** materials. The students imagine the tens frames to move around dots as they solve problems:
   8 + 5, 3 + 9, 8 + 9.
6. Link the doubles to the Make a Ten strategy.

   Ask the students to consider all the ways they know to work out 8 + 9. The range of likely responses is:

   - Double 8 + 1 = 16 + 1 = 17
   - Double 9 – 1 = 18 – 1 = 17
   - Double 10 – 1 – 2 = 20 – 1 – 2 = 19 – 2 = 17 (Rarely used)
   - 10 + 8 = 18 but this is 1 too many so the answer is 17
   - Remove 1 from the 8 and add 1 to the 9 and to give 7 + 10 which is 17
   - Add 2 to the 8 and remove 2 from the 9 and to give 10 + 7 which is 17

   Students work out and discuss the variety of strategies to work out: 6 + 8, 9 + 7, 6 + 9

7. Attention now turns to using Make a Ten strategies to solve subtraction problems. Subtraction problems with the first number in the teen decade are given.
   Michael has 13 sweets and he eats 5 of them sweets. How many sweets does Michael now have now?

   Students model 13 on pre-printed ten frames.

Two equally good methods are likely.
EITHER remove the 3 to leave the 10 then remove a further 2 to leave 8.
OR remove 5 from the 10 to leave 5 and add the 3 to give 8.

8. Students use pre-printed tens frames to solve number stories for subtraction problems. Use: 14 - 6, 17 - 8, 12 - 8, 17 - 5, 13 - 5, 13 - 7, 14 - 3.

   Notice the presence of subtractions like 17 - 5. They prevent the mindset that may develop that make a ten is always appropriate. In fact, in this example, breaking the ten is not necessary because 7 - 5 = 2 and add the 10 back in gives 12.

9. Takeaways by adding.
   Charlotte thinks of a way to work out 13 - 9 by adding. She asks "9 and what makes 13?" and comes up with the answer 4.

   Ask the students how they think Charlotte does this. (She has gone 9 + 1= 10, she remembers the 1, she goes 10 + 3 = 13 and adds the 1 to the 3 to get 4.)

10. Uses Charlotte's addition method to work out these:
    12 – 9, 14 – 8, 17 – 14, 16 - 7
11. Choosing the best way:
    Use either an addition and a subtraction way to work these:
    13 - 9, 13 - 4, 17 - 15, 17 - 2, 19 - 17 , 12 – 8.

Reflecting

At the end of each session encourage the students to share and discuss answers as a group or class.

Home Link:

Dear Family

At school this week we have solved addtion and subtraction problems using a strategy called "Making Tens". You can help your child by asking them to explain their thinking as they solve the problems on this sheet.

Making Tens

- To work out 9 + 7 Moana removes 1 from the 7 to leave 6 and adds this 1 to the 9 to make 10. She then knows 9 + 7 = 10 + 6 so the answer is 16.

Use Moana's method of making a ten to work out these:

$9 + 6 = \boxed{}$	$7 + 9 = \boxed{}$	$8 + 6 = \boxed{}$
$4 + 8 = \boxed{}$	$15 - 7 = \boxed{}$	$13 - 6 = \boxed{}$

- To work out 9 + 8 Williams knows a number ways

1. His first way is 9 + 9 = 18 and remove 1 to give 17
2. His second way is to remove 1 from the 9 to leave 8 then add 8 and 8 to give 16, then put the 1 back to give 17.
3. His third way is to remove 2 from the 9 to leave 7, then add 7 and 7 to give 14 then add on the 2 to give 16

Write stories to show **two** different ways to work out the following:
9 + 7

Story One

Story Two

15 - 9

Story One

Story Two

Related Resources:

Figure it out Links

Some links from the Figure It Out series which you may find useful are:

Level 2-3, Number Sense and Algebraic Thinking, Book One, The No Name Game, page 22.

Level 2-3, Number, Putting Numbers to Work, page 2.

Level 2-3, Number, Knocking over Subtraction, page 12.

Level 2, Number, Book One, Birthday Time, page 10

Level 2, Number, Book Two, On and off the Train, page 14

Link, Number, Book One, King of the Castle, page 15.

Link, Number, Book One, Firewood Fever, page 16.

Level 2: The Numbers get Larger

http://www.nzmaths.co.nz/resource/numbers-get-larger

The Numbers Get Larger

Purpose:

This unit follows naturally from the Smart Doubling and Make a Ten units.
In this unit students are encouraged to develop part/whole addition and subtraction mental methods with multi-digit numbers.

Achievement Objectives:
Achievement Objective: Use simple additive strategies with whole numbers and fractions.
Specific Learning Outcomes:

use mental strategies for addition and subtraction problem that use part/whole strategies for multi-digit problems.

use the most efficient mental strategy for a given problem.

Description of mathematics:

In this unit students, are encouraged to extend part/whole methods with small numbers to multi-digit numbers.

Examples of more advanced "part/whole" methods using make a ten:

- The students work out 38 + 39 by removing 1 from the 38 to leave 37, add this 1 to the 39 to make 40 then add 37 and 40 to give 77.
- The students work out 38 + 39 by 30 + 30 = 60, 8 + 9 = 17, 60 + 17 = 77
- The students work out 38 + 39 by 40 plus 40 = 80 and then remove 2 (to get to 38) and remove a further 1 (to get to 39). So 80 - 2 - 1 = 87 gives the answer.

Students will need to have successfully understood the part/whole methods in the Make a Ten unit before attempting this unit.

Required Resource Materials:
Tens Frames (Material Master 4-6)
Activity:

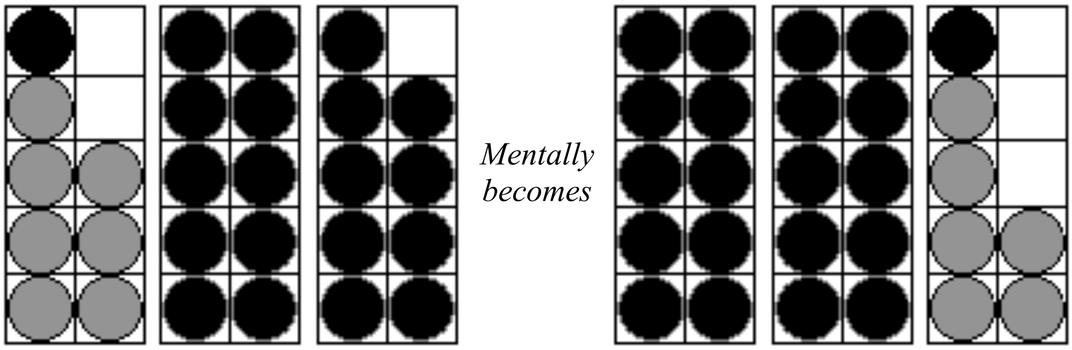
Getting Started

In this addition and subtraction unit initially only one of the numbers is 2 digits or more, while the other number is normally a single digit. This is because the processing load for the students in having both numbers with multi-digit is much higher. When the students have successfully coped with these problems 2 digit plus or minus 2 digit problems can be introduced.

Pose the problem:
Minnie has $8 and her grandmother gives her $19 for her birthday. How much money does Minnie have now?

Students model this on pre-printed tens frames.

Discuss the dots that could be moved to make the addition "easy". One example is:



(Because pre-printed tens frames are used the dots don’t actually move.)

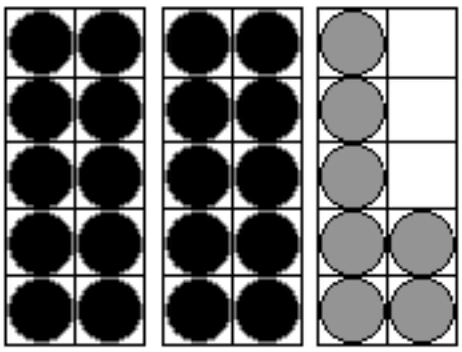
4. Repeat with similar problems, sharing the Make to Ten strategies used.

Exploring

Over the next 3-4 days pose addition and subtraction problems for the students to work on, individually, in pairs or in a small teaching group. Encourage the progression from using pre-printed tens frames to imagining the tens frames and then to completing the problem mentally.

1. Pose the problem:
   Malcolm has $27 and he spends $9 on lunch. How much does he have left?
   Students model this on pre-printed tens frames.

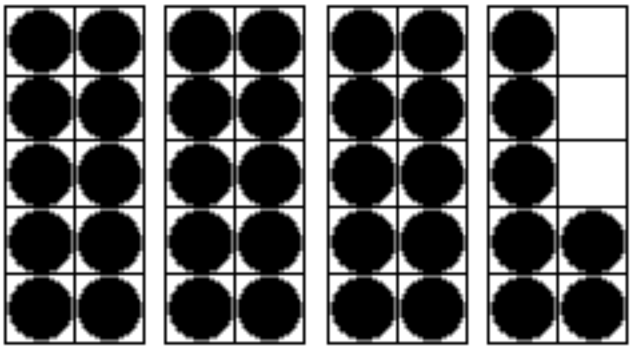
   Share solutions. Students are likely to imagine the 7 is removed leaving 20, then 2 is removed from one of the tens to leave 18. Other methods are possible.

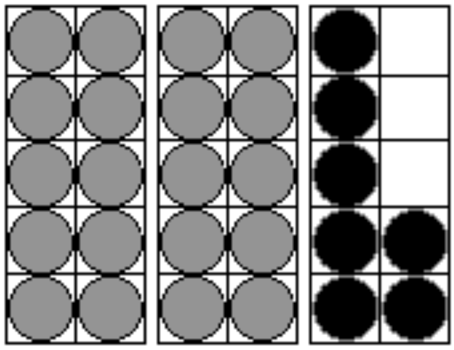


2. Pose the following problems for the students to work on. Give the students pre-printed tens frames to help in their solutions:
   28 + 7, 33 - 6, 24 - 5, 38 - 6, 24 + 9, 33 + 5, 40 - 6

3. Pose the following problems. For these problems encourage the students to imagine the tens frames:
 38 + 6, 23 – 8, 44 – 5, 39 – 9, 9 + 34, 3 + 45, 30 – 9
4. When the students are able to solve problems involving a single and double-digit numbers extend the problems to involve two 2-digits numbers. Have pre-printed tens frames available.

Pose the problem:
Betty has 37 Jaffas and 26 wine gums. How many sweets does Betty have altogether?

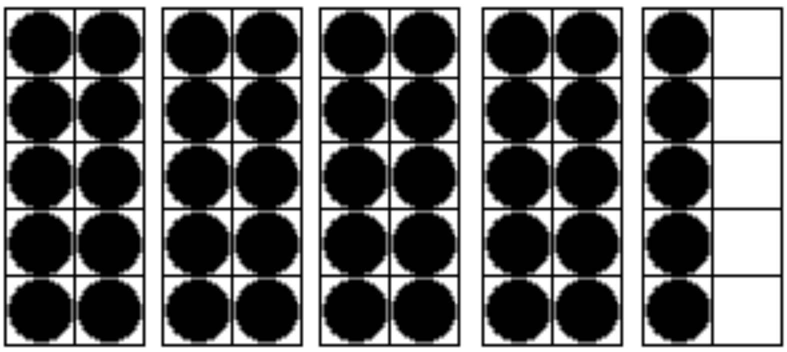




Discuss the students's ideas for solving the problem.

- One strategy involves putting the tens together to get 50 sweets and then combining the 7 + 6 to get 13. The 13 is then added to the 50 to get 63.
- Alternatively they may combine the tens to give 50, then put the 9 with the 50 to give 59, then use previous part/whole thinking to add 3 to the 57 to give 60 then add the remaining 3 to give 63.

5. Pose problems that involve the use of doubles. As with single-digit problems doubling strategies are useful for certain problems. For example:
24 + 25 (25 + 25 = 50, then subtract 1)
15 + 16, 35 + 36, 24 + 26, 97 + 103, 402 + 398
6. Pose the double-digit subtractions problems, for example:
Bernice has $45. She buys a top for $29. How much does she have left?



Discuss the ideas that the students have for solving this problem. Strategies may include:

- take away 20 from the 45 to leave 25. Another 9 needs to be removed. Remove the 5 to leave 20 then remove 4 more to give an answer of 16.
- take away 20 from the 45 to leave 25. Another 9 needs to be removed. Remove 9 from a 10 frame to leave 1. There is another 10 and a 5 to add to 1 to give 16.
- take away 30 from the 45 to leave 15. But taking away 30 is 1 too many. So add 1 to 15 to give 16.

7. Pose problems for the students to work on.
   47 – 28, 50 – 27, 100 – 68, 91 – 12, 63 – 23, 42 – 38, 103 – 6, 103 – 98, 81 – 34,
   200 - 188
8. Extras for experts.

   In these problems the students are encouraged to add and subtract more than 2 numbers in which there are smart ways to do them. Begin by discussing the following problem:

   Harry adds up the number of pies his class order in the week. By pairing up some of the numbers Harry quickly noticed the total pies sold for the week was 60. How did Harry work this answer out so quickly?

Monday	28
Tuesday	17
Wednesday	2
Thursday	3
Friday	10

   (Pairing the 28 with the 2 gives 30. Pairing the 17 with the 3 gives 20: 30 + 20 + 10 = 50.)

9. Molly has $56 when she goes shopping. She buys a hat for $28 and a pair of shoes for $26. To work out how much money she has left over she writes down 56 - 28 - 26. (Write this expression on the board.
   Her friend Kate sees this and almost immediately says Molly has $2 left. How did Molly do this so quickly?

   Discuss the students' ideas:

   - 56 - 26 = 30, 30 - 28 = 2
   - 28 + 26 = 54 (from 27 + 27) and 56-54 =2

10. Ask the students to find "clever" ways to work out the following problems:

198 + 65 + 2	345 - 99 - 245	88 + 45 + 12 + 55
100 - 34 - 66	9 + 456 + 191	7 + 25 + 33 + 25

Reflecting

At the end of each session gather the students together to share strategies.

Home Link:

Dear Family

At school this week we have explored clever strategies for working out the answers to addition and subtraction problems

Mental Magic for Experts

Lindy knows a number of ways to work out 19 + 38

- Her first way is 20 + 40 = 60. Now 20 is one more than 19 and 40 is 2 more than 38 so take away 3 from 60 to leave 57.
- Her second way is to remove add 10 and 30 to give 40, then add 9 and 8 to give 17 then add 40 and 17 to give 57.
- Her third way is to remove 2 from the 19 to leave 17, then add this 2 to the 38 to give 40 then add and 7 to give 14 then add 17 and 40 to give 57

  Write stories to show two different ways to work out the following problems. Make sure that both stories produce the same answer.

  58 + 27

  81 - 19

Figure it Out Links

Some links from the Figure It Out series which you may find useful are:

Level 2-3, Number Sense and Algebraic Thinking, Book One, Wrapping up Wontons, page 1.

Level 2-3, Number Sense and Algebraic Thinking, Book One, 50 First, page 4.

Level 2-3, Number Sense and Algebraic Thinking, Book Two, Make 28, page 14.

Level 2-3, Number Sense and Algebraic Thinking, Book Two, Tidying Up, page 16.

Level 2-3, Number, Maps and Magic, page 10.

Level 2-3, Number, Alien Addition, page 11.

Level 2, Number, Book One, Hip Hup Hop, page 8.

Level 2, Number, Book One, Weka Wobble, page 11.

Link, Number, Book One, Archery Addition, page 1.

Link, Number, Book One, Absolutely Abseiling, page 19.

Link, Number, Book Two, More Problems, page 15.

Level 3: Partitions

http://www.nzmaths.co.nz/resource/partitions

Partitions

Purpose:

This unit is about partitioning whole numbers. It focuses on partitioning numbers to "make a ten" or a decade when adding whole numbers, for example 8 + 6 can be solved as (8 + 2) + 4. The unit uses measurement as a context.

Achievement Objectives:

Achievement Objective: Generalise that the next counting number gives the result of adding one object to a set and that counting the number of objects in a set tells how many.

Achievement Objective: Partition and/or combine like measures and communicate them, using numbers and units.

Specific Learning Outcomes:

partition numbers less than 10

know and use "teen" facts

solve addition problems by making a ten, or making a decade

solve addition problems involving measurements

Description of mathematics:

Students at Level Two should understand that numbers are counts that can be split in ways that make the operations of addition, subtraction, multiplication and division easier. From Level One students will understand that counting a set tells how many objects are in the set. At Level Two they are learning that the count of a set can be partitioned and that the count of each subset tells how many objects are in that subset. Students also need to understand that partitions of a count can be recombined. For example, a count of ten can be partitioned into 1 and 9, 2 and 8, 3 and 7, etc.

It is important that students understand that because there are many ways to partition a number and they will need to choice the best partition to suit the question. In this unit we focus on making partitions that allow numbers to make a ten. For example 8 + 6, the 6 can be partitioned to 2 + 4, the 2 then combines with 8 to make a ten 8 + 2, + 4. For this reason teachers should ensure that they select questions that are best solved using the "make a ten" strategy as opposed to partitioning to make doubles. For example, 8 + 7 also encourages the strategy 7 + 7 + 1 rather than 8 + 2 + 5.

This unit uses the context of measurement. Examples and problems should use like measures, and students should be encouraged to write the units when they give an answer, for example 8m + 5m = 13m.

Required Resource Materials:

Plastic animals

Unifix cubes
Tens frames Material Master 4-6 (available from Material Masters)
Small bottle/s
Ice cube tray/s
Numeral flip strip Material Master 4-2 (available from Material Masters)
Packet of 10 items, and single items (e.g pens)
Activity:

Session 1

In this session students investigate the fact that whole numbers can be partitioned in a number of ways.

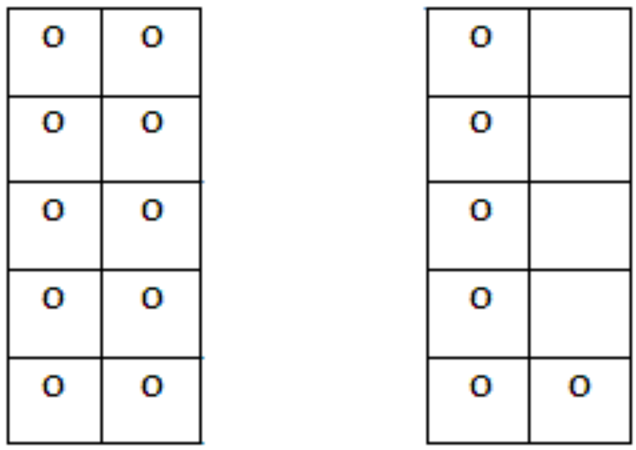
1. Show the students a bag of 8 plastic farm animals and a piece of paper with 2 rectangles drawn to represent 2 paddocks. Tell the students you are going to find all the ways of splitting the animals between the 2 paddocks.
2. Ask the students:
   How many animals should I put in the first paddock? (Put that number (e.g 3) in that paddock and the rest in the second paddock)
   How many animals are in the second paddock? Make sure the students understand there is still 8 animals.
3. Record for the students 3 + 5 = 8.
4. Continue working with the students to find all the pairs of number that add to 8. For example, 6 + 2 and 2 + 6.
5. Show the students a length of 7 unifix cubes. Work with students to find different ways of partitioning the 7 cubes. Record the partitions, for example 4 + 3.
6. Students can continue to explore partitioning numbers for numbers up to 10. Students can work in pairs or individuals to partition the number and record the results. Students can use unifix cubes, sets of counters, strips of squared paper as materials to help. Students who know the basic addition and subtraction facts to 10 will be able to partition numbers without materials.

Session 2

In this session students investigate that partitioning teen numbers using the number ten. It is easiest to start by making teens numbers as 10 + x.

1. Show the students a packet of pens (or item that comes in packs of 10) and 5 single pens.
2. Ask the students: *how many pens do I have?* (Students may count on 11, 12, 13 etc,)
3. Record the answer as 10 + 5 = 15. Do several more examples.
4. Show the students two tens frames, one complete with 10 dots and another with 6 dots. Ask the students how many dots are there? Again record the answer as 10 + 6 = 16. Using the tens frames work with the students to do solve more examples.
5. Students also need to be able to understand that teens numbers can be partitioned. Tell the students that in today’s session the numbers are going to be split into 10 and something.

6. Show the students two blank teens frame and a pile of 16 counters. Tell the students you know there are more than 10 counters in the pile, but how many more? Put the counters on the tens frames.



7. Ask the students: *16 can be split into 10 plus what?*
8. Record the answer as 16 = 10 + 6.
9. Name some objects in the classroom that are between 11 and 20 cm. Ask the students to measure the length using their ruler and record the answer on the table.

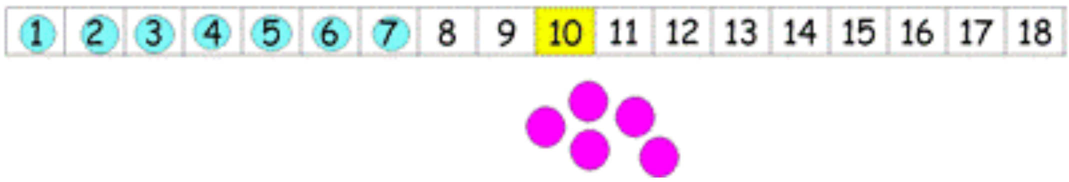
Object	Length	Splits
Sellotape dispenser		10 cm + cm
Duster		
Stapler		
Notebook		
etc		

10. Other measurement contexts can be used to provide practice activities. For example, capacity. Pour enough water into a bottle to make about 18 ice cubes. Ask the students to pour the water into the ice cubes tray and count how many ice cubes it would make. Ask the students to write their answer as 10 + ? Bottles with different amounts of water can be used.

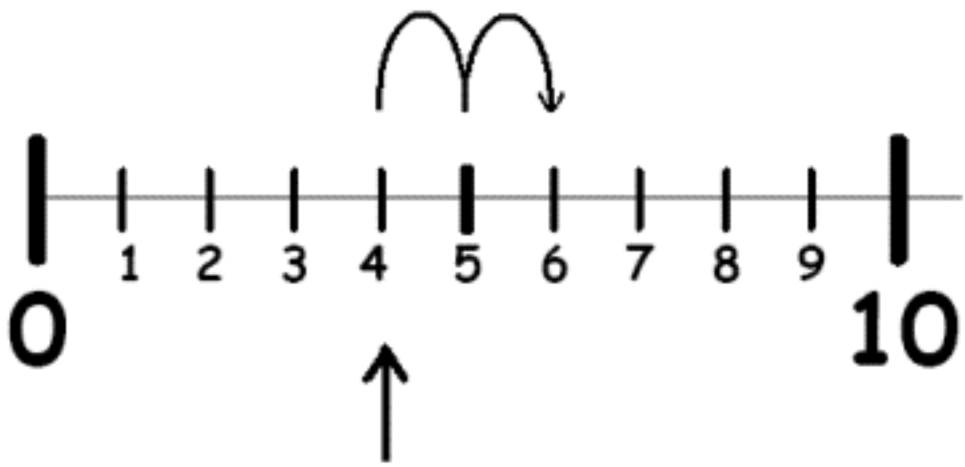
Session 3

In this session students solve addition problems by partitioning numbers. They use the strategy "make a ten".

1. Show the students a number strip (see Resources) from 0 – 20 and colour in the 10 square. Place 7 counters on the strip. Tell the students that you are going to use the number strip to help add 7 + 5. Show the students a group of 5 counters.



2. Ask the students: *how many counters will it take to get to 10?* (3)
   Put on the 3 counters then ask the students: *how many counters are there to add on?* (2)
   What does 10 and 2 make?
   What two numbers did we split the 5 into? (3 +2)
   Why did we chose 3, then 2? (To make it up to 10)
3. Write the problem 8 + 6 for the students to see.
4. Ask the students: *how many counters would be need to get to 10? (2)*
   How many counters are there still to add on? (4)
   What is 10 plus 4? (14)
   Check the answer using counters and the number strip.
5. *Draw a number line and pose the question 7 + 4.*
   Start at the 7, ask the students: how many jumps do we need to add on? (4)
   How many jumps is it to get to 10? (3)
   How many left over from the 4? (1)
   What is 10 and 1? (11)



6. Students can practise using the make a ten strategy on number lines. Pose questions using the context of measurement and encourage students to write the correct units beside the answer. Possible questions are:
   - The bucket had 9 litres in it and George poured in another 6 litres. How many litres are now in the bucket?
   - Anna put 7 cups of juice on the tray and Pam added another 5 cups to the tray. How many cups were there altogether?
   - The temperature was 8°C and it rose 3 degrees during the morning. What is the temperature now?

- James left George's house and ran 8 minutes then walked for 5 minutes before he got home. How many minutes did it take him to get home?
- Mum bought 7 kilograms of potatoes and 4 kilograms of carrots. How much did the vegetables weigh?

Session 4

In this session students solve addition problems by partitioning numbers. They will solve problems that involve adding a 1 digit number to a 2 digit number. The "make a ten" strategy is applied to decade numbers, for example 28 + 7 is solved by making it up to 30 and then adding the remaining 5.

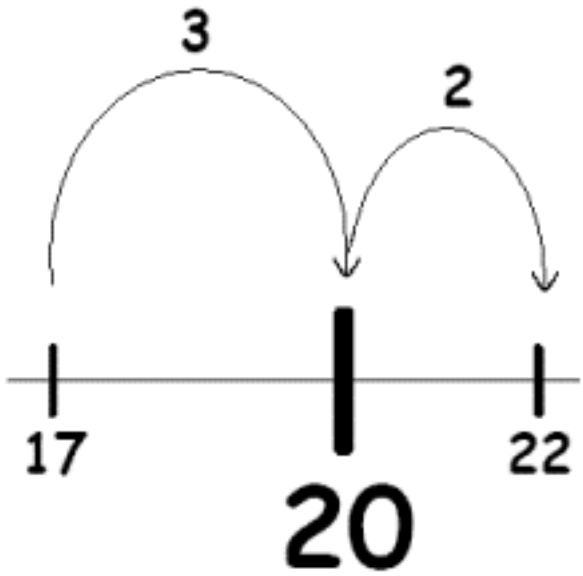
1. Pose the problem: If the plant was 37cm tall and it grew 8cm, how tall is it now? Show the students a number line that ranges from 0 – 100, with the 10s numbers coloured in.
2. Ask the students: *Using the make a ten strategy we used yesterday, how could we solve this problem?*

   1.

3. Work with students to jump 3 to get to 40, then jump the remaining 5 to get to 45.

   2.

4. Pose the question: The suitcase weighed 17 kilograms and the backpack weighed 5 kilograms. How much did the luggage weigh altogether?
   Show the students how they can draw a number line to suit the question.
   Ask the students : *what numbers are we adding together?* (17 and 5)
   Draw a line and the number 17 underneath.
   Ask the students: *what number can be jump to from here?* (20)
   how many jumps is that? (3)
   how many of the 5 are left? (2)
   What is the answer? (22)



5. Students can practise using the make a decade strategy on their own number lines. Pose questions using the context of measurement and encourage students to write the correct units beside the answer. Possible questions are:
   - Jane was going home on the bus. The bus took 26 minutes then she walked for 8 minutes. How long did it take for Jane to get home?
   - Peter's plant was 48cm tall. It grew another 7cm. How tall is Peter's plant now?
   - Dad went to the garden shop and bought 16kg of compost and 7kg of fertilizer. How much did it all weigh?
   - The painter had 65 litres of paint and he bought another 8 litres. How much paint does he now have?
   - In the first three weeks of October Wellington had 88mm of rainfall, in the rest of the month another 7mm fell. How much rain is that altogether?

Session 5

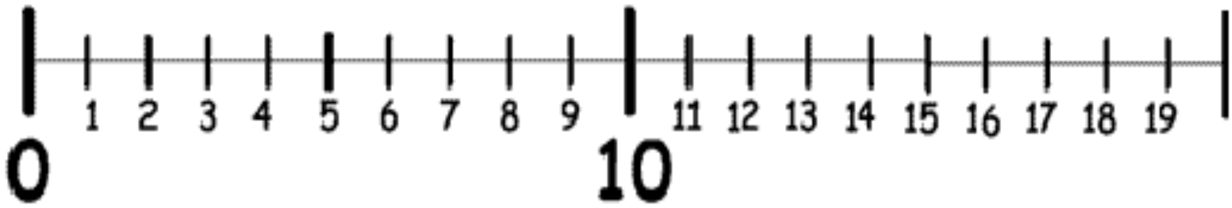
In this session you may wish to continue to give the students opportunities to practise addition partitioning with the make a ten or make a decade strategy. Problems could focus around one measurement theme, for example length. Or problems could focus around a theme such as camping and involve more than one measurement context, for example, weight of packs, time spent on activities, capacity of shower water, length of washing lines, etc.

Alternatively you may wish to use the formats of session 3 and 4 to show students how this partitioning strategy can be used to solve subtraction problems. For example, 44 – 7, it takes 4 jumps to get back to 40, then the remaining 3 jumps takes us back to 37.

Home Link:

Dear Family,

This week your child has been learning to use a number line and jumps to 10 to work out addition questions. Ask them to show you how they jump to the 10 to solve 8 + 5.



Level 3: Addition & Subtraction Pick & Mix
http://www.nzmaths.co.nz/resource/addition-and-subtraction-pick-n-mix?parent_node=

Addition and Subtraction Pick n Mix

Purpose:

In this unit we look at a range of strategies for solving addition and subtraction problems with whole numbers with a view to students anticipating from the structure of a problem which strategies might be best suited.

Achievement Objectives:
Achievement Objective: Use a range of additive and simple multiplicative strategies with whole numbers, fractions, decimals, and percentages.
Specific Learning Outcomes:

mentally solve whole number addition and subtraction problems using:

- compensation from tidy numbers including equal additions
- place value
- reversibility

use appropriate recording techniques

predict the usefulness of strategies for given problems

evaluate the effectiveness of their selected strategies

generalise the types of problems that are connected with particular strategies

Description of mathematics:

The strategy section of the New Zealand Number Framework consists of a sequence of global stages that students use to solve mental number problems. On this framework students working at different strategy stages use characteristic ways to solve problems. This unit of work is useful for students working at stage 6 of the Number Framework, Advanced Additive. Students at this stage select from a broad range of strategies to solve addition problems, subdividing and recombining numbers to simplify problems. The Number Framework also includes a knowledge component which details the knowledge students will need to develop in order to progress through the strategy stages of the framework. This unit draws on students' knowledge of compatible numbers to 10 and 100, and place value relationships to 1000.

The key teaching points are:

- Some problems can be easier to solve in certain ways. Teachers should elicit strategy discussion around problems in order to get students to justify their decisions about strategy selection in terms of the usefulness of the strategy for the problem situation.
- Tidy number strategies are useful when number(s) in an equation are close to an easier number to work from.
- When applying tidy numbers to addition, the total or sum must remain unchanged.
- When applying tidy numbers to subtraction, the difference between numbers must remain unchanged.
- Place value strategies are useful when no renaming is needed.
- Reversibility strategies are useful for subtraction problems where place value and numbers will be ineffective.

Required Resource Materials:
Place value equipment
Activity:

Getting Started

The purpose of this session is to explore the range of strategies that students have to solve addition and subtraction problems. This will enable you to elicit the strategies that students currently use to solve addition and subtraction problems, evaluate which strategies need to be focused on in greater depth and identify students in your group as "expert" in particular strategies. There are two problems given as examples for exploration. You may want to use further examples of your own.

Problem 1: Sarah has $288 in the bank. She then deposits her pay cheque for $127 from her part time job at PetCare. How much does she have now?

Ask the students to work out the answer in their heads. Give the students 2-3 minutes thinking time. Then ask them to share their solutions and how they solved it with their learning partner. The following are possible responses:

Place value:
288 + 127 is just like 288 + 100 +20 +7. So that's 388…408…415.

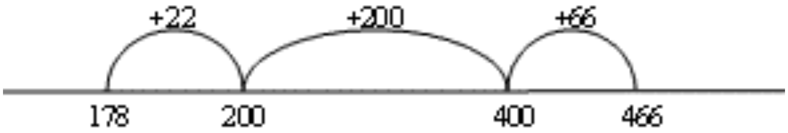
Tidy numbers:
If I tidy 288 to 300 it would be easier. To do that I need to add 12 to 288, which means I have to take 12 off the 127. So that's 300 plus 115. Easy!

As different strategies arise ask the students to explain why they chose to solve the problem in that way. Accept all the strategies that are elicited at this stage, recording them to reflect upon later in the unit (perhaps in a modelling book).

Problem 2 Sarah has $466 in her bank account and spends $178 on a new mp3 player, how much money does she have left in her bank account?

Ask students to solve the problem mentally, giving them 2-3 minutes thinking time. Then ask them to share their solutions and how they solved it with their learning partner. Possible responses are:

Reversibility:
$466 - $178 is the same as saying how much do you need to add to $178 to get $466. $178 plus $22 makes $200, plus $200 more makes $400 plus $66 makes $466. If you add up $22 plus $200 plus $66 you get $288.



Tidy numbers using equal additions:
You round the $178 to $200 by adding $22. $466 - $200 is $266. Then you put on $22 to keep the difference the same, so it's $288.

There is a strong possibility that some students may misapply the addition tidy numbers strategy to subtraction problems. When using tidy numbers in subtraction the difference between the numbers in the equation must remain the same, so if you add an amount to one number, you must also add it to the other number. This is the opposite to addition, where the sum of the two numbers must remain unchanged when using a tidy numbers strategy, meaning that if you add an amount to one number you must subtract it from the other number. Some students may solve subtraction problems like 63 – 28 by tidying 28 to 30 (adding two), and 63 to 61 (subtracting two). The student has failed to keep the difference unchanged.

In this situation the students' beliefs may need to be challenged by posing a conflict situation, using a simple example. For example, misapplying the tidy numbers strategy to 20 - 8 will leave the students with the problem 18 - 10. The situation can be modelled using Cuisenaire rods or a ruler.

As different strategies arise ask the students to explain why they chose to solve the problem in that way. Accept all the strategies that are elicited at this stage, recording them to reflect upon later in the unit (perhaps in a modelling book).

Ask students to reflect on the strategies that have been discussed in the session and evaluate which strategies that they personally need further work on, perhaps using thumb signals - thumbs up - confident and competent with the strategy, thumbs sideways - semi confident, thumbs down - not yet confident.

Use this information to plan for your subsequent teaching from the exploring section outlined below.

Exploring

Over the next two to three days, explore the following strategies making explicit the strategy you are concentrating on as the teacher and the reason for using the selected strategy

e.g. *In the problem 357 + 189 tidy numbers would be a useful strategy because 189 is close to 200.*

Each day follow a similar lesson structure to the introductory session, with students sharing their solutions to the initial questions and discussing why these questions lend themselves to the strategy

being explicitly taught. Conclude each session by having students making some statements about when this strategy would be useful and why e.g. tidy numbers when one number is close to 100 or 1000, place value when no renaming is needed and reversibility when neither of the other two are helpful for subtraction. It is important to record these key ideas as they will be used for reflection at the end of the unit.

The questions provided are intended as examples for the promotion of the identified strategies. If the students are not secure with a strategy you may need to make up some of your own questions to address student needs.

Tidy numbers/Equal additions
Room 9 are selling muesli bars at lunchtime to raise money for their camp. They had 434 at the beginning of lunchtime and sold 179, how many did they have left to sell?

The tidy numbers strategy involves rounding a number in a question to make the question easier to solve. In the above question, 179 can be rounded to either 180 (by adding 1), or 200 (by adding 21). Tidying one number will affect the other numbers in an equation. For subtraction questions, it is important that the difference between numbers remains the same. In the question above, the number to be tidied is 179 (to 200). In order to do this, we add 21. To keep the difference between the numbers the same, we must add 21 to the number we are subtracting from. The net result of the equation is then:
(434 + 21) - (179 + 21) =
455 – 200 (the tidied number) =
255

For addition questions, the amount used to 'tidy' needs to be taken from the other number(s) in the equation. The net result of tidying the below question (739 + 294) is:
(294 + 6) + (739 – 6) =
300 (the tidied number) + 733 =
1033

Lead the students to discover the effect that tidying numbers has on the quantities you are dealing with. The following questions can be used to elicit discussion about the strategy.

- *What tidy number could you use that is close to one of the numbers in the problem?*
- *What do you need to do to the other number if you tidy up this number? Why?*
- *Why is this strategy useful for this problem?*
- *What knowledge helps you to solve a problem like this?*

If the students do not understand the tidy numbers concept, use place value equipment (for example a number line) to show the problems physically. Some students may find it useful to record and keep track of their thinking.

Use the following questions for further practice if required:
Martha has scored 739 runs for her cricket club this season. Last season she scored 294, how many did she score in total in the last two seasons?
Farmer Dan has 1623 sheep and he sells 898 sheep at the local sale. How many sheep does he have left?
568 + 392
661 - 393
1287 + 589
1432 - 596

Note that the problems posed here are using a tidying up strategy rather than tidying down i.e. 103 down to 100 as in these situations place value tends to be a more useful strategy.

Place Value For the community hangi 356 potatoes had been peeled and there were 233 left to be peeled, how many potatoes will there be altogether?

The place value strategy involves adding the ones, tens, hundreds, and so on. In the above problem:
300 + 200 is added
Then 50 + 30
And finally 6 + 3

The following questions can be used to elicit discussion about the strategy:

- *How can you use your knowledge of place value to solve this problem?*
- *Why is this strategy useful for this problem?*

If the students do not understand the concept, use place value equipment (such as blocks) to show the problems physically. Some students may find it useful to record and keep track of their thinking .

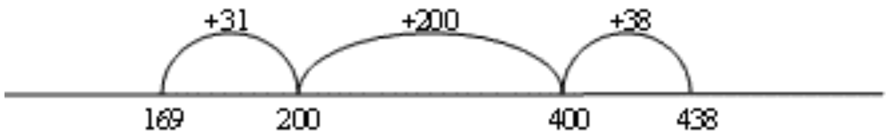
Use the following questions for further practice if required:
Zac has $498 on his eftpos card and spends $243 on a new BMX bike, how much money does he have left?
3221 + 348
4886 - 1654
613 + 372
784 - 473

Reversibility

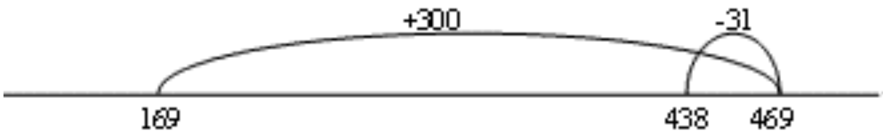
Faloa is helping his Mum build a fence. There were 438 bricks in the pile and they used 169 of them yesterday, how many bricks have they got left for today?

The reversibility strategy involves turning a subtraction problem into an addition one so the problem above becomes:
169 + ? = 438

Then using tidy numbers to solve



Or



The following questions can be used to elicit discussion about the strategy:

- *How could we think of this as an addition equation?*
- *What do you need to add to make it easier to solve?*
- *How can you keep track of how much you have added altogether?*
- *Why is this strategy useful for this problem?*
- *What knowledge helps you to solve a problem like this?*

If the students do not understand the concept, use a number line to show the problems physically. Some students may find it useful to record and keep track of their thinking using a number line.

Use the following questions for further practice if required:
At the mail sorting office there were 547 letters to be sorted, 268 of these were distributed to private boxes, how many were left to be delivered?
The school library has a total collection of 1034 books and 459 are issued at the moment, how many are on the shelves?
628 - 342
537 - 261
742 - 353
1521 - 754
1762 - 968
1656 - 867

Reflecting

As a conclusion to the week's work, give the students the following five problems to solve asking them to discuss which strategy they think will be useful for each problem and why before they solve them. After they have solved the problems, engage in discussion about the effectiveness of their selected strategies for the problems.

There may be a few students who do not concur with the group about the usefulness of a particular strategy for a given problem. This is perfectly acceptable as long as they are able to justify their thinking. Many students will have a favourite strategy that they use, sometimes to the exclusion of all others. Most often this is a place value strategy. These students should not be discouraged from using place value to solve problems but should be exposed to problems where place value is an inefficient strategy, because of repeated renaming, for example 289 + 748, or 453 – 257.

Problems for discussion
1318 - 747
763 - 194
433 + 452
1993 + 639
4729 - 1318

You might also like to also try some problems with more than 2 numbers in them
721 – 373 - 89
663 - 61 - 88
63 + 422 + 49
42 + 781 + 121
84 + 343 - 89

Discuss the different strategies explored during the week and ask students to explain in their own words what types of problem each strategy would be useful for solving, and what types of problem each strategy would not be useful for solving. Ask the students to draw a strategy 'from a hat' and write questions specific to that strategy for a partner.

Conclude the unit by showing the students the questions asked in the initial session again and discuss whether they would solve them in a different way now, and how their thinking has evolved.

Level 4: Multiplication & Division Pick & Mix 1
http://www.nzmaths.co.nz/resource/multiplication-and-division-pick-n-mix-1?parent_node=

Multiplication and Division Pick n Mix 1

Purpose:

In this unit we look at a range of strategies for solving multiplication and division problems with whole numbers with a view to students anticipating from the structure of a problem which strategies might be best suited. This unit builds on the ideas presented in the Multiplication Smorgasbord lesson (Numeracy Book 6, page 52).

Achievement Objectives:

Achievement Objective: Use a range of multiplicative strategies when operating on whole numbers.
Achievement Objective: Generalise properties of multiplication and division with whole numbers.

Specific Learning Outcomes:

mentally solve whole number multiplication and division problems using:

- proportional adjustment
- place value partitioning
- rounding and compensation
- factorisation

use appropriate recording techniques

predict the usefulness of strategies for given problems

evaluate the effectiveness of selected strategies

generalise the types of problems that are connected with particular strategies

Description of mathematics:

The strategy section of the New Zealand Number Framework consists of a sequence of global stages that students use to solve mental number problems. On this framework students working at different strategy stages use characteristic ways to solve problems. This unit of work is useful for students working at stage 7 of the Number Framework, Advanced Multiplicative. Students at this stage select from a broad range of strategies to estimate and solve multiplication and division problems, subdividing and recombining numbers to simplify problems either additively or multiplicatively. The Number Framework also includes a knowledge component which details the knowledge students will need to develop in order to progress through the strategy stages of the framework. This unit draws on students' knowledge of multiplication and related division facts to 10, whole number place value and connection of basic facts to multiplying powers of ten.

The key teaching points in this unit are:

- Some problems are easier to solve in certain ways. Teachers should elicit strategy discussion around problems in order to get students to justify their decisions about strategy selection in terms of the usefulness of the strategy for the problem situation.
- Useful strategies for multiplication include place value partitioning, rounding and compensating, proportional adjustment and factorisation.
- Useful strategies for division include proportional adjustment (with factorisation), rounding and compensating, and partitioning or 'chunking'.
- Tidy number strategies (rounding and compensating) are useful when number(s) in an equation are close to an easier number to work from.
- When applying tidy numbers in multiplication and division it is important to keep track of what has been changed in a problem in order to compensate (rounding and compensating).
- Place value strategies are most useful when little or no renaming is needed.
- Proportional adjustment is useful when there is a connection between the numbers that can be used to simplify the problem such as doubling and halving or quadrupling and quartering. Division with factorisation can be viewed as a form of proportional reasoning. In division both of the numbers must be reduced by the same factor.
- Factorisation is useful for multiplication when one of the factors can be reduced.

Required Resource Materials:
Large dotty array - Material Master 6-9 (128KB)
Place value equipment- beans, place value blocks
Activity:

Getting Started

The purpose of this session is to explore the range of strategies that students have to solve multiplication and division problems. This will enable you to elicit the strategies that students currently use and evaluate which strategies need to be focused on in greater depth as well as identifying students in your group as "expert" in particular strategies.

Problem 1:
Craig bikes 38 kilometres each day for five days. How many kilometres has he travelled by the end of the five days?

Ask students to work out the answer in their head. Give the students 2-3 minutes thinking time. Then ask them to share their solutions and how they solved it with their learning partner. The following are possible responses:

Rounding and compensating:
38 x 5
38 is rounded to 40 so the equation becomes 40 x 5 then 10 which is (2 x 5) is subtracted

Proportional adjustment:
38 x 5
Solve instead 19 x 10 using doubling and halving (by halving 38 and doubling 5)

Place value partitioning:
38 x 5
Solve 30 x 5, add 8 x 5

As different strategies arise ask the students to explain why they chose to solve the problem in that way. Accept all the strategies that are elicited at this stage, recording them to reflect upon later in the unit (perhaps in a modelling book).

Problem 2:
There were 136 rowers entered in the eights rowing champs at the Maadi Cup. How many teams are entered?

Ask the students to work out the answer in their heads. Give the students 2-3 minutes thinking time. Then ask them to share their solutions and how they solved it with their learning partner. The following are possible responses:

<u>**Place value partitioning (chunking):**</u>
136 ÷ 8
I know that 36 ÷ 8 = 4 with a remainder of 4
I now have 104 ÷ 8, I know that 10 x 8 = 80 and that leaves me 24 which is 3 groups of 8 so the answer is 13 and the first four which is 17.

<u>**Factorisation (proportional adjustment):**</u>
Dividing by 8 is like dividing by 2 then 2 then 2 so half 136 is 68 and half 68 is 34 and divide by 2 again leaves me 17 so the answer is 17.

<u>**Rounding and compensating:**</u>
160 would be the same as 8 times 20. 136 is 24 less than that, or three teams less than 20… it's 17.

As different strategies arise ask the students to explain why they chose to solve the problem in that way. Accept all the strategies that are elicited at this stage, recording them to reflect upon later in the unit (perhaps in a modelling book).

Ask students to reflect on the strategies that have been discussed in the session and evaluate which strategies that they personally need further work on, perhaps using thumb signals - thumbs up - confident and competent with the strategy, thumbs sideways - semi confident, thumbs down - not yet confident. Use this information to plan for your subsequent teaching from the exploring section outlined below.

Exploring

Over the next two to three days, explore the following strategies making explicit the strategy you are concentrating on as the teacher and the reason for using the selected strategy.

e.g. *In the problem 29 x 7 tidy numbers would be a useful strategy as 29 is close to 30.*

The following questions are provided as examples for the promotion of the identified strategies. If the students are not secure with a strategy you may need to make up some of your own questions to address student needs.

<u>**Rounding and Compensating (Multiplication)**</u>
The Sting netball fans are going to Christchurch to watch a netball game. Each bus has 48 people on it and there are 14 buses travelling altogether. How many Sting fans are heading to Christchurch?

The rounding and compensating strategy involves rounding a number in a question to make it easier to solve. In the above question 48 can be rounded to 50 (by adding 2). The problem then becomes 50 x 14, or 700. In order to compensate for the rounding, two lots of 14 (28) must be subtracted from the 'rounded' equation.

The following questions can be used to elicit discussion about the strategy:

- *What tidy number could you use that is close to one of the numbers in the problem?*
- *What do you need to do if you tidy up this number? Why?*
- *Why is this strategy useful for this problem?*
- *What knowledge helps you to solve a problem like this?*

If the students do not seem to understand the tidy numbers concept, use place value equipment or a large dotty array to show the problems physically. Some students may find it useful to record and keep track of their thinking.

Use the following questions for further practice if required:
69 x 9
148 x 7
398 x 6
26 x 32
7 x 9998
548 x 3

Note that the problems posed here are using a tidying up strategy rather than tidying down. If one of the factors is just over a tidy number (such as 203) then place value tends to be a more useful strategy.

<u>Rounding and compensating (Division)</u>
Sarah uses eight bus tickets every week to travel around town. She wins 152 tickets in a radio competition. How long will they last her?
Rounding and compensating for division involves finding a number that is close to the total, and working from that number to find an answer. For the question above, a student might say:
I know that 8 times 20 would be 160. 152 is 8 less than 160, so the tickets would last her 19 weeks.

If the students do not seem to understand the rounding and compensating concept, use a large dotty array to show the problems physically. Some students may find it useful to record and keep track of their thinking.
343 ÷ 7
198 ÷ 9
1194 ÷ 6
686 ÷ 7
1764 ÷ 18

<u>Proportional Adjustment (Multiplication)</u>
At the music festival there are 32 schools with 25 students in each choir, how many students are there altogether in the choirs?
Proportional adjustment involves using knowledge of multiples to create equivalent equations. Factors are proprtionally adjusted to make one (or both) factors easier to work from. In the above problem the factors could be adjusted as follows:

$$\begin{array}{lcl} 32 & \times & 25 \\ \downarrow \div 4 & & \downarrow \times 4 \\ 8 & \times & 100 \end{array}$$

Or, using doubling and halving:

$$\begin{array}{lcl} 32 & \times & 25 \\ \downarrow \div 2 & & \downarrow \times 2 \\ 16 & \times & 50 \end{array}$$

The following questions can be used to elicit discussion about the strategy:

- *What could you multiply one of these numbers by to make it easier to work with?*
- *What would you then need to do to the other number?*
- *Why is this strategy useful for this problem?*
- *What knowledge helps you to solve a problem like this?*

If the students do not seem to understand the proportional adjustment concept, use a large dotty array to show the problems physically. Some students may find it useful to record and keep track of their thinking.

Use the following questions for further practice if required:
333 x 18 (thirding and trebling)
60 x 750
300 x 180 (thirding and trebling)
120 x 225
24 x 125

Proportional Adjustment (Division)
Volunteers from the Southland Ornithological Society tagged 1680 sooty shearwaters in a 12 month period. How many on average did they tag per month?
In division, proportional adjustment involves changing both numbers in the equation by the same factor. Therefore, the numbers used to proportionally adjust the problem must be factors of both numbers in the equation. For example:
If I divide the 1680 and the 12 by 2 my equation becomes 840 ÷ 6 and I can divide them both by 2 again to get 420 ÷ 3 which is 140. Or I could divide them both by 4 to get the same equation..

The following questions can be used to elicit discussion about the strategy:

- *What could you divide both of these numbers by to make an easier equation?*
- *Why is this strategy useful for this problem?*
- *What knowledge helps you to solve a problem like this?*

If the students do not seem to understand the proportional adjustment concept, use equipment to show the problems physically. Some students may find it useful to record and keep track of their thinking.

Use the following questions for further practice if required:
1800 ÷ 15 (→ 3600 ÷ 30)
1962 ÷ 18 (→ 981 ÷ 9)
1498 ÷ 14 (→ 749 ÷ 7)
1728 ÷ 16 (→ 864 ÷ 8)

Place Value Partitioning (Multiplication)
Nick has $3121, and needs 8 times this amount to buy the new four wheel drive he wants. How much money does the four wheel drive cost?
The place value strategy involves multiplying the ones, tens and hundreds. In the above problem the student might say the following:
I multiplied 3000 x 8 and got 24 000 then I added the $800 (100 x 8) and 160 (20 x 8) then added the 8 (1 x 8) to get 24 968

The following questions can be used to elicit discussion about the strategy:

- *How can you use your knowledge of place value to solve this problem?*
- *Why is this strategy useful for this problem?*

If the students do not seem to understand the partitioning concept, show the problems physically. Some students may find it useful to record and keep track of their thinking. An extension of the place value strategy involves the use of standard written form for multiplication.

Use the following questions for further practice if required:
61 323 x 30
7 x 4110
1020 x 40
342 x 11

<u>Place value partitioning (division)</u>
Pisi has an after school job at the market, bagging pawpaw into lots of 6. If there are 864 pawpaw to be bagged, how many bags can he make?
The place value partitioning strategy for division involves 'chunking' known facts and subtracting them from the answer. The long division written form will be familiar to most teachers. In the case above, a student might think:
100 lots would be 600. That leaves me with 264. I can take 120 away from that, which is twenty lots of 6. That leaves 144. If I take another 120 I get 24, which is 4 lots of 6. So I've taken away 100 lots, then 20 then 20, then 4… the answer's 144.

This thinking could be recorded as:

6) 864	
-600	100
264	
-120	20
144	
-120	20
24	
-24	4
	144

If the students do not seem to understand the partitioning concept, show the problems physically. Some students may find it useful to record and keep track of their thinking. An extension of the place value strategy involves the use of standard written form for division.

Use the following questions for further practice if required:
676 ÷ 4
9760 ÷ 8
3808 ÷ 7
3472 ÷ 15
2546 ÷ 18

<u>Factorisation (Multiplication and Division)</u>
Stephanie has 486 marbles to share evenly amongst eighteen of her friends. How many marbles will each person get?

The factorisation strategy involves using factors to simplify the problem. In this instance eighteen can be factorised as 2 x 3 x 3. This means dividing by two, then three, then three has the same net effect as dividing by 18. Likewise, multiplying by two, then three, then three has the same net effect as multiplying by 18. In applying factorisation to the above problem, a student might think:

18 is the same as 2 x 3 x 3. So I have to halve 486, then third, then third. If I divide 486 by 2 I get 243. 240 divided by 3 is 80, so 243 divided by 3 will be 81. 81 divided by 3 is 27. The answer is 27.

The following questions can be used to elicit discussion about the strategy:

- *How can you use your knowledge of factors to solve this problem?*
- *Why is this strategy useful for this problem?*

If the students do not understand the factorisation concept, show the problems physically. Some students may find it useful to record and keep track of their thinking.

Use the following questions for further practice if required:
532 ÷ 8 (÷2, ÷2, ÷2)
348 ÷ 12 (÷2, ÷2, ÷3)
4320 ÷ 27 (÷3, ÷3, ÷3)
135 x 12 (x2, x2, x3)
43 x 8 (x2, x2, x2)
27 x 16 (x2, x2, x2, x2)

Each day follow a similar lesson structure to the introductory session, with students sharing their solutions to the initial questions and discuss why these questions lend themselves to the strategy being explicitly taught. Conclude each session by having students make some statements about when this strategy would be useful and why (e.g. "place value is useful when there is limited renaming required" or "factorisation is useful when one of the factors is able to be renamed as a series of smaller factors"). It is important to record these key ideas as they will be used for reflection at the end of the unit.

Reflecting

As a conclusion to the weeks work, give the students the following five problems to solve asking them to predict which strategy they think will be useful for each problem and why they think this is the most useful strategy before they solve them. After they have solved the problems engage in discussion about the effectiveness of their selected strategies for the problems.

When discussing there may be a few students who do not concur with the group about the usefulness of a particular strategy in a given problem. This is perfectly acceptable as long as they are able to provide a reasonable justification for their thinking.

Problems for discussion (more than one strategy might be suitable for these)
48 x 50 (proportional adjustment)
559 ÷ 13 (place value partitioning)
29 x 16 (rounding and compensating)
1926 ÷ 18 (proportional adjustment)
212 x 11 (place value partitioning)
704 ÷ 8 (factorisation)
153 ÷ 17 (rounding and compensating)
421 x 8 (factorisation)

Ask the students to create problems for a partner where one of the strategies covered in this unit is the most useful.

Conclude the unit by showing the students the questions asked in the initial session again and discuss whether they would solve them in a different way now, why or why not. Review the modelling book or record of statements or generalisations about the strategies and make any changes.

Level 5: Multiplication & Division Pick & Mix 2
http://www.nzmaths.co.nz/resource/multiplication-and-division-pick-n-mix-2?parent_node=

Multiplication and Division Pick 'n' Mix 2

Purpose:

In this unit we look at a range of strategies for solving multiplication and division problems with whole numbers and decimal fractions, with a view to students anticipating from the structure of a problem which strategies might be best suited. This unit builds on the ideas presented in Multiplication and Division Pick 'n' Mix 1.

Achievement Objectives:

Achievement Objective: Understand operations on fractions, decimals, percentages, and integers.

Specific Learning Outcomes:

mentally solve decimal fraction multiplication and division problems using:

- proportional adjustment
- place value
- tidy numbers
- factorisation

use appropriate recording techniques

predict the usefulness of strategies for given problems

evaluate the effectiveness of selected strategies

generalise the types of problems that are connected with particular strategies

Description of mathematics:

The strategy section of the New Zealand Number Framework consists of a sequence of global stages that students use to solve mental number problems. On this framework students working at different strategy stages use characteristic ways to solve problems. This unit is useful for students working at stage 8 of the Number Framework, Advanced Proportional. Students at this stage select from a broad range of strategies to estimate and solve multiplication and division problems involving decimal fractions. The Number Framework also includes a knowledge component which details the knowledge students will need to develop in order to progress through the strategy stages of the framework. This unit draws on students' knowledge of multiplication and related division facts to 10, compatible decimal fractions to 1 (along with whole number compatibility) and place value relationships to 3 decimal places.

The key teaching points in this unit are:

- Some problems are easier to solve in certain ways. Teachers should elicit strategy discussion around problems in order to get students to justify their decisions about strategy selection in terms of the usefulness of the strategy for the problem situation.
- Useful strategies for multiplication include place value partitioning, rounding and compensating, proportional adjustment and factorisation.
- Useful strategies for division include proportional adjustment (with factorisation), rounding and compensating, and partitioning or ‘chunking’.
- Tidy number strategies (rounding and compensating) are useful when number(s) in an equation are close to an easier number to work from.
- When applying tidy numbers in multiplication and division it is important to keep track of what has been changed in a problem in order to compensate (rounding and compensating)
- Place value strategies are most useful when little or no renaming is needed.
- Proportional adjustment is useful when there is a connection between the numbers that can be used to simplify the problem such as doubling and halving or quadrupling and quartering. Division with factorisation can be viewed as a form of proportional reasoning. In division both of the numbers must be reduced by the same factor.
- Factorisation is useful for multiplication when one of the factors can be reduced.

Required Resource Materials:
Decimal place value equipment (e.g. decimats)
Activity:

Getting Started

The purpose of this session is to explore the range of strategies that students have to solve multiplication and division problems. This will enable you to elicit the strategies that students currently use and evaluate which strategies need to be focused on in greater depth as well as identifying students in your group as "expert" in particular strategies.

Problem 1:
Sharon swims 0.5 kilometres each day for 28 days. How many kilometres has she travelled by the end of the five days?

Ask students to work out the answer in their head. Give the students 2-3 minutes thinking time. Then ask them to share their solutions and how they solved it with their learning partner. The following are possible responses:

Rounding and Compensating:
28 x 0.5
28 is rounded to 30, so the equation becomes 30 x 0.5 (15). Then 2 x 0.5 (1) must be removed to complete the equation.

Proportional adjustment:
28 x 0.5
Solve instead 14 x 1 - doubling and halving

Place value partitioning:
28 x 0.5
Solve (20 x 0.5) + (8 x 0.5).

As different strategies arise ask the students to explain why they chose to solve the problem in that way. Accept all the strategies that are elicited at this stage, recording them to reflect upon later in the unit (perhaps in a modelling book).

Problem 2:
Ruataniwha House are selling bags of nuts for a fundraiser. They buy a sack of nuts weighing 27kg, and repackage the nuts into 0.45 kg bags for sale. How many bags will come from the sack? (27 kg ÷ 0.45 kg)

Ask the students to work out the answer in their heads. Give the students 2-3 minutes thinking time. Then ask them to share their solutions and how they solved it with their learning partner. The following are possible responses:

<u>Place Value Partitioning (Chunking):</u>
27 ÷ 0.45
10 bags would make 4.5 kg, so 20 bags would make 9 kg. 3 lots of 9 is 27… they can make 60 bags.

<u>Proportional adjustment:</u>
If I double both sides I get 54 ÷ 0.9. 54 ÷ 9 is 6, so 54 ÷ 0.9 must be 60.

As different strategies arise ask the students to explain why they chose to solve the problem in that way. Accept all the strategies that are elicited at this stage, recording them to reflect upon later in the unit (perhaps in a modelling book).

Ask students to reflect on the strategies that have been discussed in the session and evaluate which strategies that they personally need further work on, perhaps using thumb signals - thumbs up - confident and competent with the strategy, thumbs sideways - semi confident, thumbs down - not yet confident. Use this information to plan for your subsequent teaching from the exploring section outlined below.

Exploring

Over the next two to three days, explore the following strategies making explicit the strategy you are concentrating on as the teacher and the reason for using the selected strategy e.g.

e.g. *In the problem 2.9 x 7 rounding and compensating would be a useful strategy as 2.9 is close to 3.*

The following questions are provided as examples for the promotion of the identified strategies. If the students are not secure with a strategy you may need to make up some of your own questions to address student needs.

<u>Rounding and compensating (Multiplication)</u>
Bozo the Clown fires 4.8 L of water from his water pistol each night at the circus. How many litres does he fire over two weeks?
The tidy numbers strategy involves rounding a number in a question to make it easier to solve. In the above question 4.8 can be rounded to 5 (by adding 0.2). The problem then becomes 5 x 14. The 14 groups of 0.2 L added to 'tidy' the problem now need to be subtracted, leaving a total of 67.3 L.

The following questions can be used to elicit discussion about the strategy:

- *What tidy number could you use that is close to one of the numbers in the problem?*
- *What do you need to do if you tidy up this number? Why?*
- *Why is this strategy useful for this problem?*
- *What knowledge helps you to solve a problem like this?*

If the students do not understand the rounding and compensation concept, use place value equipment to show the problems physically. Some students may find it useful to record and keep track of their thinking.

Use the following questions for further practice if required:
6.9 x 9
1.48 x 7
13.98 x 6
12.96 x 32
7 x 9.998
5.48 x 3

Note that the problems posed here are using a tidying up strategy rather than tidying down. If one of the factors is just over a tidy number (such as 203) then place value tends to be a more useful strategy.

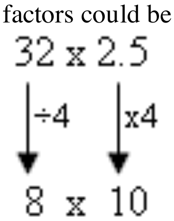
Rounding and compensating (Division)
Ohau House uses 0.7L of detergent every time they do a fundraising car wash. They have 13.3L of detergent left… how many car washes will that last?
Rounding and compensating for division involves finding a number that is close to the total, and working from that number to find an answer. For the question above, a student might say:
I know that 0 .7 times 20 would be 14. 13.3 is 0.7 less than 14, so the detergent would last 19 car washes.

The following questions can be used to elicit discussion about the strategy:

- *What tidy number could you use that is close to one of the numbers in the problem?*
- *What do you need to do if you tidy to this number?*
- *Why is this strategy useful for this problem?*
- *What knowledge helps you to solve a problem like this?*

If the students do not seem to understand the rounding and compensation concept, show the problems physically. Some students may find it useful to record and keep track of their thinking.
34.3 ÷ 7
19.8 ÷ 9
119.4 ÷ 6
13.3 ÷ 0.7
1683 ÷ 1.7

Proportional Adjustment (Multiplication)
Bob the bodybuilder adds 2.5 kg to his weightlifting bar each day. How much will he have added after 32 days?
Proportional adjustment involves using knowledge of multiples to create equivalent equations. Factors are proprtionally adjusted to make one (or both) factors easier to work from. In the above problem the factors could be adjusted as follows:



Alternatively, students might double the 2.5 to 5, and halve 32 to 16.

The following questions can be used to elicit discussion about the strategy:

- *What could you multiply one of these numbers by to make it easier to work with?*
- *What would you then need to do to the other number?*
- *Why is this strategy useful for this problem?*
- *What knowledge helps you to solve a problem like this?*

If the students do not understand the proportional adjustment concept, use place value equipment to show the concept physically. Some students may find it useful to record and keep track of their thinking.

Use the following questions for further practice if required:
3.33 x 18 (thirding and trebling)
60 x 7.5
300 x 1.8 (thirding and trebling)
120 x 2.25
24 x 1.25

Proportional Adjustment (Division)
Jonno has 168m of rope to cut into 1.2m lengths. How many bits of rope can he make?
In division proportional adjustment involves reducing or increasing both numbers in the equation by the same number. Therefore, the numbers used to proportionally adjust the problem must be factors of both numbers in the equation. For example,
If I divide the 168 and the 1.2 by 2 my equation becomes 84 ÷ 0 .6 and I can divide them both by 2 again to get 42 ÷ 0 .3 which is 140 or I could divide them both by 4 to get the same equation.

The following questions can be used to elicit discussion about the strategy:

- *What could you divide both of these numbers by to make an easier equation?*
- *Why is this strategy useful for this problem?*
- *What knowledge helps you to solve a problem like this?*

If the students do not understand the proportional adjustment concept, use equipment to show the problems physically. Some students may find it useful to record and keep track of their thinking.

Use the following questions for further practice if required:
180 ÷ 1.5 (→ 360 ÷ 3)
367.5 ÷ 3.5 (→ 735 ÷ 7)
196.2 ÷ 18 (→ 98.1 ÷ 9)
1498 ÷ 1.4 (→ 749 ÷ .7)
172.8 ÷ 16 (→ 86.4 ÷ 8)

Place Value Partitioning (Multiplication)
Mae Ling uses 3.12m to make a traditional dance outfit. How much fabric will she use to make 8 outfits?
The place value strategy involves multiplying in place value (e.g. ones, tenths and hundredths). In the above problem the student might say the following:
I multiplied 3 x 8 and got 24. Then I added the 0.8 (0.1 x 8) and 0.16 (0.02 x 8) to get 24 .96 m

The following questions can be used to elicit discussion about the strategy:

- *How can you use your knowledge of place value to solve this problem?*
- *Why is this strategy useful for this problem?*

Use the following questions for further practice if required:
613.23 x 30
7 x 4.1112

10.21 x 40
354 x 0.11

If the students do not understand the partitioning concept, use place value equipment to show the problems physically. Some students may find it useful to record and keep track of their thinking. An extension of the place value strategy involves the use of standard written form for multiplication.

<u>Place value partitioning (division)</u>
Sheila mixes 0.8kg of milk powder with water each time she feeds the calves. If there is 49.6kg left in the bag, how many feeds will the milk powder last?
The place value partitioning strategy for division involves 'chunking' known facts and subtracting them from the answer. The long division written form will be familiar to most teachers. In the case above, a student might think:
Ok, 10 lots would be 8kg. That means 50 lots would be 40kg. That leaves me with 9.6 kg. So if I take off another 10 lots that's another 8kg. That leaves me with 16 kg left over, or 2 lots. So the answer is 50, plus 10, plus 2… 62 feeds!

This thinking could be recorded as:

0.8) 49.6	
-40.0	50
9.6	
-8.0	10
1.6	
-1.6	2
	62

The following questions can be used to elicit discussion about the strategy:

- *How can you use your knowledge of place value to solve this problem?*
- *Why is this strategy useful for this problem?*

If the students do not understand the partitioning concept, use place value equipment to show the problems physically. Some students may find it useful to record and keep track of their thinking. An extension of the place value strategy involves the use of standard written form for division.

Use the following questions for further practice if required:
67.6 ÷ 0.4
97.6 ÷ 0.8
380.8 ÷ 0.7
472 ÷ 1.5
546 ÷ 1.8

<u>Factorisation (multiplication and division)</u>
Farmer Betty is erecting a 148.8m long fence. She places a post every 8m. How many posts does she need?

The factorisation strategy involves using factors to simplify the problem. In this instance eight can be factorised as 2 x 2 x 2. This means dividing by two, then two, then two has the same net effect as dividing by 8. Likewise, multiplying by two, then two, then two has the same net effect as multiplying by 8. In applying factorisation to the above problem, a student might think:
Dividing by 8 is the same as dividing by 2, then 2, then 2. So, 148.8 ÷ 2 = 74.4. Then 74.4 ÷ 2 = 37.2. And last, 37.2 ÷ 2 = 18.6. The answer is 18.6m.

The following questions can be used to elicit discussion about the strategy:

- *How can you use your knowledge of factors to solve this problem?*
- *Why is this strategy useful for this problem?*

If the students do not seem to understand the factorisation concept, show the problems physically. Some students may find it useful to record and keep track of their thinking.

Use the following questions for further practice if required:
53.2 ÷ 8 (÷2, ÷2, ÷2)
3.48 ÷ 12 (÷2, ÷2, ÷3)
43.2 ÷ 27 (÷3, ÷3, ÷3)
1.35 x 12 (x2, x2, x3)
4.3 x 8 (x2, x2, x2)
2.7 x 16 (x2, x2, x2, x2)

Each day follow a similar lesson structure to the introductory session, with students sharing their solutions to the initial questions and discuss why these questions lend themselves to the strategy being explicitly taught. Conclude each session by having students make some statements about when this strategy would be useful and why (e.g. "place value is useful when there is limited renaming required" or "factorisation is useful when one of the factors is able to be renamed as a series of smaller factors"). It is important to record these key ideas as they will be used for reflection at the end of the unit.

Reflecting

As a conclusion to the weeks work, give the students the following five problems to solve asking them to predict which strategy they think will be useful for each problem and why they think this is the most useful strategy before they solve them. After they have solved the problems engage in discussion about the effectiveness of their selected strategies for the problems. When discussing there may be a few students who do not concur with the group about the usefulness of a particular strategy in a given problem. This is perfectly acceptable as long as they are able to provide a reasonable justification for their thinking.

Problems for discussion (more than one strategy might be suitable for these)
68 x 3.5 (proportional adjustment)
46.2 ÷ 1.4 (place value partitioning)
2.93 x 6 (rounding and compensating)
169.5 ÷ 1.5 (proportional adjustment)
23.2 x 11 (place value partitioning)
7.04 ÷ 8 (factorisation)
161.7 ÷ 1.63 (rounding and compensating)
4.11 x 16 (factorisation)

Ask the students to create problems for a partner where one of the strategies covered in this unit is the most useful.

Conclude the unit by showing the students the questions asked in the initial session again and discuss whether they would solve them in a different way now, why or why not. Review the modelling book or record of statements or generalisations about the strategies and make any changes.